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Tracking Gain Expression of a Strong Signal-to-Noise Monopulse Radar

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This paper is concerned with the signal-to-noise ratio for a four-horn type monopulse radar. The complexity of such a system lies in the nonlinearity of the system which consists of two nonlinear devices with a linear device between them. A simple expression for gain is obtained for the strong signal-to-noise case under the assumption that the look angle is small.

Introduction

THE system under consideration is a four-horn type monopulse-radar-signal process used for angle tracking. It is well known that phase determines the target angle. In practice, the receiver Doppler radar consists of limiters, a Doppler filter, a discriminator, and a phase detector. In addition to the signal an amount of clutter noise is introduced into the system. The problem is to determine the signal-to-noise ratio for such a system. Here, it is limited to the strong signal-to-noise case. The complexity of such an analysis lies in the nonlinearities of the system. Since limiters are nonlinear devices, the system under consideration consists of two nonlinear devices with a linear device, a Doppler filter, between them. It would be interesting to compare this system with that consisting of two linear devices with a nonlinear device between them, as considered by Kac and Siegert.[1]

Since this is a Doppler system, the output of the first (ideal) limiter is expressed in terms of a Fourier expansion.[2] A mean-value type approximation is used here. It is also assumed that the "look angle" is small. Both assumptions can be justified for the system under consideration. A simple expression for the gain is obtained for the strong signal-to-noise case. Some results are plotted, for gain vs signal-to-noise ratio.

Brief Review

The far-field antenna pattern in a plane is given by the analytical function

$$g(\theta) = \int_{-l}^{l} h(x) e^{ikx\sin\theta} \mathrm{d}x$$

where $h(x)$ is an amplitude complex-valued distribution function, k is the wavelength, θ is the look angle, and l is the slot width.

$$h(x) = |h(x)| \exp iq(x)$$

then $|g(\theta)|$ is a maximum at

$$\theta = \sin^{-1}\left(-\frac{q(x)}{kx}\right), \qquad 0 \le \theta \le \pi$$

This θ is called the look angle at which $g(\theta)$ has maximum amplitude.

Let $g_1(\theta)$ and $g_2(\theta)$ be two returning signals from g_1 with $0 \le x \le l$ and g_2 with $-l \le x \le 0$, respectively; see Fig. 1.

The receiver horns are a distance $d = 2l$ apart. If the target is in line with the direction of the look angle, then the two returning signals have no phase difference. Otherwise, if the target makes an angle θ with the direction of the look angle, a phase difference exists which can be easily computed. It is

$$2\cos(\tfrac{1}{2}kd\sin\theta)\exp(ik\tfrac{1}{2}d\sin\theta)$$

This establishes the relationship between θ and the phase difference. Hence the target direction is determined by measuring the phase difference.

So far the discussion has been limited to a plane. The discussion to extend this analysis to a three-dimensional space is obvious. The target direction is determined by two phase differences, that is, with a four-horn type system. In practice, the returning signal goes through a set of devices for processing, and noise is introduced by the target background. This analysis is presented next.

Analysis

The angle tracking process under consideration is a four-horn type Doppler radar. The signal processing system consists of two linear devices (see Fig. 2). The inputs are at i.f. frequency and mixed with clutter noise. The inputs may be considered as consisting of three parts:

Sum signal	$A\cos(\omega_{\mathrm{i.f.}}t + \phi(t))$
Azimuth difference signal	$\mu A\sin(\omega_{\mathrm{i.f.}}t + \phi(t))\cos\omega_m t$
Elevation difference signal	$\gamma A\sin(\omega_{\mathrm{i.f.}}t + \phi(t))\sin\omega_m t$

where A is the amplitude of the signal of the radar return at the carrier frequency; μ the azimuth to sum amplitude ratio of the signal; γ the elevation to sum amplitude ratio of the signal; $\phi(t)$ the phase of radar return at the carrier frequency; $\omega_{\mathrm{i.f.}}$ the carrier frequency of the sum channel; and ω_m the lobing or multiplex frequency.

A, μ, and γ are treated as constants since they change very slowly as the target moves. The clutter-noise contributions in the sum, azimuth-difference, and elevation-difference channels are assumed to be mutually independent, narrow, and Gaussian. They can be expressed, respectively, as

$$N_j(t) = x_j(t)\cos\omega_m t + y_j(t)\sin\omega_m t, \qquad j = 1,2,3$$

and have the properties

$$E\{x_j^2(t)\} = E\{y_j^2(t)\} = E\{N_j^2(t)\}, \qquad j = 1,2,3$$

where E denotes statistical expectation. Let

$$p_j = A^2/2E(x_j^2), \qquad j = 2,3$$

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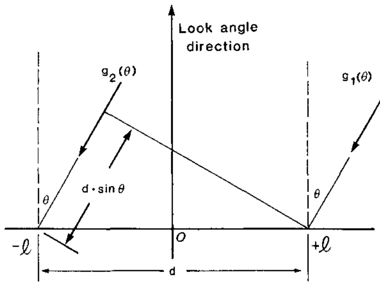

Fig. 1 Two returning signals from g_l.

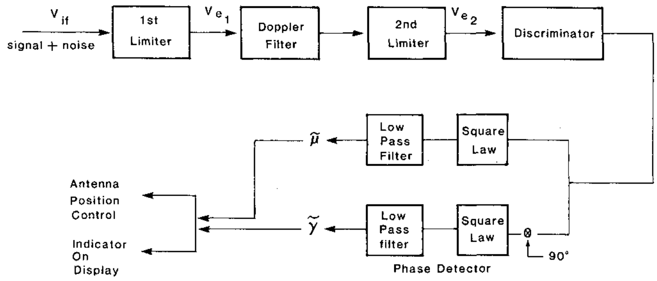

Fig. 2 The signal processing system.

It is assumed that

$$p_j \gg 1, \qquad j=2,3$$

That is, the system has a strong signal-to-noise ratio.

The objective of this paper is to compute the "gain expression"—the ratio of the expressed measured angle corrupted by the clutter noise by considering the strong signal-to-noise case.

The composite i.f. signal at the input to the first (ideal) limiter is

$$V_{\text{i.f.}} = \alpha(t)\sin[l_{\text{i.f.}}x+\phi(t)] + \beta(t)\cos[\omega_{\text{i.f.}}+\phi(t)]$$

$$= [\alpha^2(t)+\beta^2(t)]^{1/2}\cos[\omega_{\text{i.f.}}+\phi(t)-\psi(t)]$$

where the envelope is composed of two parts:

$$\alpha(t) = \{(\mu A+y_2)^2+(\gamma A+y_3)^2\}^{1/2}\cos(\omega_m t-\zeta)+y_l$$

$$\beta(t) = (x_2^2+x_3^2)^{1/2}\cos(\omega_m t-\zeta)+A+x_l$$

with

$$\zeta = \tan^{-1}[(\gamma A+y_3)/(\mu A+y_2)]$$

and

$$\xi = \tan^{-1}(x_3/x_2)$$

and the phase

$$\Psi(t) = \tan^{-1}[\alpha(t)/\beta(t)]$$

Then the output of the first (ideal) limiter is

$$V_{ll} = \cos[\omega_{\text{i.f.}}t+\phi(t)-\psi(t)] = \cos\Psi(t)\cos[\omega_{\text{i.f.}}t+\phi(t)]$$

$$+\sin\psi(t)\sin[\omega_{\text{i.f.}}t+\phi(t)]$$

Since the noise power terms in the sum channel are small in comparison with other terms, they may be neglected. Let

$$M(t) = [\alpha(t)/\beta(t)]\cos(\omega_m t-\zeta)$$

in which the x_l and y_l terms are being dropped in the quotient. It is proper to take Fourier-series expansions of $\cos\psi(t)$ and $\sin\psi(t)$. The corresponding Fourier-series coefficients for $\cos\psi(t)$ and $\sin\psi(t)$ are

$$a_n = \frac{1}{2\pi}\int_{-\pi}^{\pi}\frac{\cos nt\,dt}{[1+M^2(t)\cos^2 t]^{1/2}}$$

and

$$b_n = \frac{1}{2\pi}\int_{-\pi}^{\pi}\frac{M(t)\cos t\cos nt\,dt}{[1+M^2(t)\cos^2 t]^{1/2}}$$

In trying to obtain such coefficients, a mathematical difficulty occurs because of the presence of the function $M(t)$. The mean-value theorem will be used to provide an approximation for $M(t)$. The mean-value theorem states that if $f(x)$ is a smooth function, then there exists a $\tilde{t}$, with $a \le \tilde{t} \le b$, such that

$$\int_a^b f(t)g(t)\,dx = f(\tilde{t})\int_a^b g(t)\,dt$$

In case the values of the function $f(x)$ are bounded in a narrow strip, then $f(\tilde{t})$ can be approximated by the expectation $E[f(x)]$. For this reason, and the assumption $p_j \gg 1$ for $j=2$ and 3, the function $M(t)$ is approximated by

$$\frac{(\mu A+y_2)^2+(\gamma A+y_3)^2}{\tfrac{1}{2}[E(x_2^2)+E(x_3^2)+2A^2]}$$

To compute the Doppler filter output it is necessary to transfer the time history function V_{ll} into the frequency domain ω_m. For this purpose a Fourier cosine expression for $\cos\psi(t)$ is applied. The Doppler filter output consists of the d.c. and the first-harmonic terms of ω_m:

$$a(t)\cos[\omega_{\text{i.f.}}t+\phi(t)] + 2b(t)\cos[\omega_m t-\zeta(t)]\sin[\omega_{\text{i.f.}}+\phi(t)]$$

where $a(t)$ and $b(t)$ are complete elliptic integrals of the first and second kinds. The output of the second (ideal) limiter is

$$V_{l2} = \cos\{[\omega_{\text{i.f.}}t+\phi(t)]-\Lambda(t)\}$$

where the phase function

$$\Lambda(t) = \tan^{-1}\{[b(t)/a(t)]\cos[\omega_m t-\zeta(t)]\}$$

To evaluate the phase function under the strong signal-to-noise assumption, the approximation

$$2b(t)/a(t) = M(t)+0(M^4(t))$$

is used where $0(M^4(t)) \to 0$ as $M^4(t) \to 0$. Finally the outputs of the phase detectors are

$$\tilde{\mu} = \tan^{-1}[k(\mu A+y_2)p^{-1}]$$

$$\tilde{\gamma} = \tan^{-1}[k(\gamma A+y_3)p^{-1}]$$

where k is a device constant and

$$p = [\tfrac{1}{2}E(x_2^2)+\tfrac{1}{2}E(x_3^2)+A^2]^{1/2}$$

In practice, the tracking angles are small (usually limited to less than 8 deg). Replace $\tan^{-1}\theta$ by θ, then

$$E(\tilde{\mu}) = \frac{k\mu A}{[\tfrac{1}{2}E(x_2^2)+\tfrac{1}{2}E(x_3^2)+A^2]^{1/2}}$$

since $E(y_2)=0$. Let $\tilde{\mu}_0$ and $\tilde{\gamma}_0$ be the two measured angles for the system without the clutter noise. The *gain suppression* of

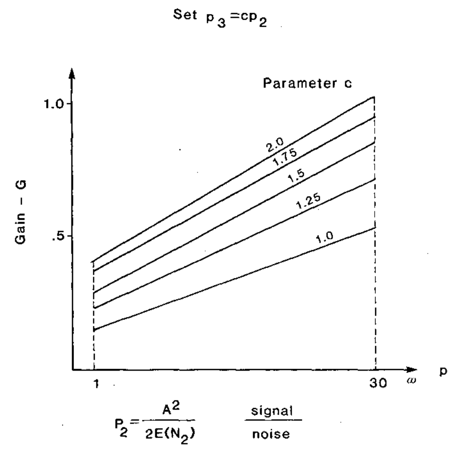

Fig. 3 $G(\theta)$ computed for various values of p_2 and c.

the system is defined as

$$G(\mu) = E(\tilde{\mu}) / E(\tilde{\mu}_0)$$

and

$$G(\gamma) = E(\tilde{\gamma}) / E(\tilde{\gamma}_0)$$

Since $E(\tilde{\theta}_0) = k\theta$, it turns out that $G(\mu) = G(\gamma)$ and

$$G(\theta) = [1 + \tfrac{1}{4}(p_2^{-1} + p_3^{-1})]^{\frac{1}{2}}$$

where

$$p_j = A^2 / 2E(x_j^2), \qquad j = 2,3$$

Based on this equation with $p_3 = p_2 c$, $G(\theta)$ has been computed for various values of p_2 and c (see Fig. 3).

Concluding Remarks

The system under consideration consists of two nonlinear devices—limiters—with a lienar device, a Doppler filter, between them. A complete analysis is a rather difficult problem. However, under the assumptions given in the analysis, the gain expression is reduced to a very simple form. The approximate result is a reasonable one from the engineering point of view in case the system involves a strong signal-to-noise ratio. An interesting extension is whether the assumption of strong signal-to-noise ratio may be removed. This may be accomplished by a filter that separates signal-to-noise either by frequency separation or by area separation.

References

[1] Kac, M. and Siegert, A.J.F., "On the Theory of Noise in Radio Receivers with Square Law Detector," *Journal of Applied Physics,* Vol. 18, 1947, p. 383.

[2] Davenport, Jr., W.B., "Signal-to-Noise in Bandpass Limiters," *Journal of Applied Physics,* Vol. 24, 1953, pp. 720-727.